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Mathematics

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Pedagogic School

The pendulum of method in arithmetical teaching has swung from the extremely concrete to the extremely abstract within the last few years. The vibrations are moreover continuing at present, but with ever-decreasing amplitude. It is to be hoped that the damping effect of mutual criticism will in the near future enable us to fix our attention upon the position of stable equilibrium.

When this golden mean has been found we shall have learned how to secure to the pupil, in an economical way, the necessary knowledge of the laws and uses of abstract number without either hide-binding his thinking with a surfeit of abstractions, assimilable only by a mature mind, or lumbering his mental movements with a useless burden of devices for "concreting number" long after their period of serviceableness has passed. We shall then be able to admit that number is a purely "mental product, put into things and not got from them," and still to see that to use things and not to be used by them number *must* be put into them, and that for the bulk of our pupils a knowledge of number and its uses is of value in direct proportion to their skill in "putting it into things."

The school can render no more valuable service to society in general to-day than to discover a way in which the boys and girls who attend may be trained into a facile working knowledge of elementary mathematics. If this way could be found, mental discipline, the inducing of judgments of magnitude, culture, important as they all are in themselves, would come as by-products in the evolution of the child into the efficient man, who would be ready, because able, to perform the service our particular time demands. Such practical mathematics is needed in our business houses, with our railroads, and in our workshops, factories, scientific laboratories, on our farms, and wherever the great work of production is in progress—most of all, in the schoolroom.

The objection quite generally made to such mathematical work in the grades as elementary surveying, questions connected with simple systems of forces, is that it is not the purpose of the public school to teach professional knowledge. This objection can hardly prevail against the argument that useful knowledge will furnish as sound a basis for culture as will useless. Such work does not seek to train the pupil for a profession, but merely to give him an insight into the reality of the need for arithmetic, algebra, and geometry in modern life; to give a view of a fuller and more vital phase of need than is furnished by the keeping of accounts and the demands of general training.

The most formidable obstacle in the way of introducing such practical work is the inability of teachers to carry it out. Most grade teachers, being women, have not been under conditions which make the stress of the mathematical needs of the industrial professions keenly felt, and are likely to infer that there is no such need

which they can supply. This situation can and will be removed in time, though time will be necessary to overcome the inertia of present conditions. An influx into the intermediate grades of a large number of strong and wide-awake men teachers would go far toward relieving the difficulties of the present situation.

To the teacher of arithmetic, it is not so much a matter of choosing between the concrete method of presentation and the abstract method, both of which may be equally artificial, as it is of choosing between the natural method, based upon approximately real conditions, and the unnatural method, based upon purely artificial conditions. The chief advantage of number to man is to enable the mind to move faster, farther, and deeper into things than it can do without it. To hold the mind bound to things too long is to crop its wings and to compel it to walk when it might fly. The mental step from things to thoughts should be taken as soon as it can be taken surely, though no sooner. It is beneath the dignity of the true teacher to become a devotee and proselyte to any method, as such.

One function of number is not ordinarily considered, or is insufficiently considered, by elementary mathematical teachers: viz., its *use in indirect measurement.* As a matter of fact, in most cases where the mathematical need cannot be evaded, a magnitude is to be ascertained which cannot for physical reasons be obtained directly. It may be a line, obstructed by a hill, a building, a body of water; it may be the radius of the earth, distance to the sun, or other heavenly body. Difficulties arising from such sources are imperative in their demands for mathematics. Some related magnitude must here be measured and the desired one inferred through some known relationship. This is the greatest of all the uses of mathematics. Problems in elementary surveying furnish a good beginning for this sort of practice. Every grade teacher, beyond the fourth, should know something and teach something in this line.

The classes will continue the topographic map through June as occasion permits.

For classroom work the following outline will guide:

I. Quantity.

1. Constant; e. g., height of a tree at a given instant.

2. Variable; e. g., height of a tree at any time.

II. Number.

1. Constant, or fixed (arithmetical number).

2. Variable.

III. Constant quantity.

1. Origin, nature, and use.

2. How it gives rise to number.

3. Possibilities of constant quantity.

(a) May be subdivided, or parted; that is, may be measured.

(b) May be used a number of times; i. e., may be used to measure another quantity.

(c) Ideas of constant number and measurement arise from these two possibilities.

IV. Measurement.

1. Counting.

2. Direct measurement.

3. Indirect measurement.

V. Operations growing out of:

1. First possibility.

(a) Subtraction; unequal parts.

(b) Division; equal parts.

2. Second possibility.

(a) Addition; unequal addends.

(b) Multiplication; equal parts.

VI. Fractions, both common and decimal.

1. How and when taught.

2. Written forms.

3. Operations upon.

VII. Longitude and time and compound numbers.

1. How and when taught.

VIII. Percentage and simple interest.

IX. Geometry and algebra in the grades.

X. Discussion of place and relation of field-work in the grades.